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Question Paper Code : 40784

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Computer Science and Engineering

MA 8351 – DISCRETE MATHEMATICS

(Common to Artificial Intelligence and Data Science/Computer Science and Business System/Information Technology)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. When do you say that two compound propositions are equivalent?
2. Define Tautology with an example.
3. Does there exists a simple graph with the degree sequence $\{3, 3, 3, 3, 2\}$?
4. State the Pigeonhole principle.
5. Define strongly connected graph.
6. Define complete graph.
7. Prove if a has inverse b and b has inverse c, then $a = c$.
8. Prove that identity element is unique in a group.
9. Define lattice homomorphism.
10. When is a lattice said to be a Boolean algebra?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) When do we say a formula is tautology or contradiction? Without constructing truth table, verify whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a contradiction or tautology. Justify your answer. (6)

(ii) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (10)

Or

(b) (i) Let m and n be integers. Prove that $n^2 = m^2$ if and only if $m = n$ or $m = -n$.

(ii) Write down the negation of each of the following statements :

(1) For all integers n, if n is not divisible by 2, then n is odd

(2) If k, m, n are any integers, where $(k-m)$ and $(m-n)$ are odd, then $(k-n)$ is even.

(3) For all real numbers x, if $|x-3| < 7$, then $-4 < x < 10$.

(4) If x is real number where $x^2 > 16$, then $x < -4$ or $x > 4$.

12. (a) (i) Use mathematical induction to show that $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$, for all non-negative integers n. (8)

(ii) State the Inclusion and Exclusion principle. Hence, using the principle, find how many faculty members can speak either French or Russian, if 200 faculty members can speak French and 50 can speak Russian, while only 20 can speak both French and Russian. (8)

Or

(b) (i) Solve the recurrence relation, $S(n) = S(n-1) + 2(n-1)$ with $S(0) = 3$, $S(1) = 1$ by finding its generating function.

(ii) Prove by mathematical induction that for every positive integer n, 3 divides $n^3 - n$.

13. (a) (i) Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete subgroup of K_5 with 4 vertices.

(ii) If $(S_1, *)$ and $(S_2, \circ)$ are two semigroups such that $f : S_1 \to S_2$ is an onto homomorphism and a relation R is defined on S, Such that $aRb \Leftrightarrow f(a) = f(b)$ for any $a, b \in S_1$ then R is a congruence relation.

Or

(b) (i) Define :

(1) Adjacency matrix and

(2) Incidence matrix of a group with examples.

(ii) Prove that any undirected graph has an even number of vertices of odd degree.

14. (a) (i) Prove that the group homomorphism preserves the identity element.

(ii) Let f be a group homomorphism from a group $(G, *)$ into a group (H, Δ) then prove that $\ker(f)$ is a subgroup. Check whether $\ker(f)$ is a normal subgroup of $(G, *)$. Justify your answer. (8)

Or

(b) (i) State and prove Lagrange's theorem.

(ii) Show that the union of two subgroups of a group G is subgroup of G if and only if one is contained in other.

15. (a) (i) Let $(L, \leq)$ be a lattice. For any $a, b \in L$, $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$.

(ii) Draw the lattice of (S, gcd, lcm) where $S = \{x : x$ is a divisor of 210$\}$.

Or

(b) (i) Show that $(N, \leq)$ is a partially ordered set where N is set of all positive integers and $\leq$ is defined by $m \leq n$ iff $n - m$ is a non-negative integer.

(ii) Prove that $(L, \wedge, \vee)$ is not a complemented lattice (under division relation) where $L = \{1, 2, 3, 4, 6, 12\}$ and also draw the Hasse diagram.